

Power allocation policies with full and partial inter-system channel state information for cognitive radio networks

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Abstract This paper investigates several power allocation policies in orthogonal frequency division multiplexing - based cognitive radio networks under the different availability of inter-system channel state information (CSI) and the different capability of licensed primary users (PUs). Specifically, we deal with two types of PUs having different capabilities: a dumb (peak interference-power tolerable) PU and a more sophisticated (average interference-power tolerable) PU. For such PU models, we first formulate two

optimization problems that maximize the capacity of unlicensed secondary user (SU) while maintaining the quality of service of PU under the assumption that both intra- and inter-system CSI are fully available. However, due to loose cooperation between SU and PU, it may be difficult or even infeasible for SU to obtain the full inter-system CSI. Thus, under the partial inter-system CSI setting, we also formulate another two optimization problems by introducing interference-power outage constraints. We propose optimal and efficient suboptimal power allocation policies for these four problems. Extensive numerical results demonstrate that the spectral efficiency achieved by SU with partial inter-system CSI is less than half of what is achieved with full inter-system CSI within a reasonable range of outage probability (e.g., less than 10 %). Further, it is shown that the average interference-power tolerable PU can help to increase the saturated spectral efficiency of SU by about 20 and 50 % in both cases of full and partial inter-system CSI, respectively.

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1 Introduction

1.1 Motivation

With the rapid growth of bandwidth hungry applications and the emergence of diverse wireless systems, the demand for spectrum has increased in recent years and is expected to grow even more in future wireless networks [1, 2]. In traditional approach of spectrum management, government agencies such as the Federal Communications Commission (FCC) in the United States regulate the spectrum allocation by exclusively allocating the frequency band to different

multiple wireless systems or operators. However, recent studies based on field measurements [3] have revealed that large portions of allocated spectrum bands are unused.

Cognitive radio has been considered as a very promising technology to efficiently utilize the scarce spectrum (inherently a limited natural resource) [4] and is also being considered in a standardization body such as IEEE 802.22 WRAN (Wireless Regional Area Network), where the white spaces of television bands can be opportunistically used [5, 6]. The term, *cognitive radio* (CR), is first introduced by Mitola [7], is a flexible and intelligent wireless technology that is aware of its surrounding environment. In spectrum sharing based CR networks, where a secondary (unlicensed) system coexists with a primary (licensed) system, a fundamental design challenge is how to maximize the throughput of secondary user (SU) while ensuring the quality of service (QoS) for primary user (PU) such as throughput, outage probability or maximum interference. Based on how not to harm the PU, transmission modes in CR networks are classified into three types: *interweaved*, *overlay* and *underlay* modes [8].

In the interweaved mode, the SU can utilize unused license bands, i.e., spectrum holes. The SU transmitter (SU-Tx) in this mode needs to have the real-time functionality for monitoring and detecting the spectrum holes that change with time and geographic location. Several spectrum-sensing techniques [9, 10] and spectrum-sharing [11, 12] strategies based on game theory have been proposed. The overlay mode enables the SU to utilize the license band even if the PU is using the band. The SU-Tx is assumed to have perfect knowledge about PU's message. Therefore, the SU-Tx may use this knowledge to mitigate the interference seen by its receiver using dirty paper coding and/or to relay the primary signal to compensate for signal-to-noise ratio at the PU receiver (PU-Rx). Devroye et al. [13] proposed a genie-aided CR channel model and derived the fundamental information-theoretical limits. In the underlay mode, simultaneous transmissions are also allowed on condition that the SU-Tx interferes with the PU-Rx below than a certain threshold, so-called *interference-temperature*. Ghasemi et al. [14] analyzed SU's capacity in fading environments, but only under a received interference-power constraint at the PU-Rx. However, since the transmit-power at the SU-Tx is also limited by hardware capabilities and safety requirements in practice, this needs to be jointly considered.

Meanwhile, OFDM (orthogonal frequency division multiplexing) technology has been adopted by most of current wireless communication systems due to its robustness to multi-path fading and the high degree of flexibility in resource allocation. While the underlay mode is usually associated with UWB (ultra wide band) and spread spectrum technologies, there are several recent works in literature [15–19] considering OFDM as a physical-layer technique in both interweave and underlay modes. In the interweave

mode, once the idle status of subchannels is identified, the resource allocation problem is almost the same as the conventional problem with the set of available subchannels. However, in the underlay mode, the resource allocation problem is completely different due to the simultaneous transmissions of the primary and secondary systems.

In this paper, we concentrate on the underlay mode and develop power allocation policies in the OFDM-based CR networks. We basically assume that intra-system channel state information (CSI) is fully available to PU-Tx and investigate the performance of several power allocation policies under the different availability of inter-system CSI to PU-Tx (i.e., both *full* and *partial* inter-system CSI). Throughout the paper, we treat the partial CSI as follows: the SU-Tx has knowledge only about the average channel gain over all the subchannels instead of individual channel gain for each subchannel. In addition, we also deal with a little considered problem so far: (i) what are the ramifications of different capabilities of the PU and (ii) how much more capacity could be obtained if the SU is operating in the same band with a *more sophisticated* (average interference-power tolerable) PU instead of a *dumb* (peak interference-power tolerable) PU.

1.2 Related work

The optimal and suboptimal power allocation algorithms in the underlay CR setting have been developed for OFDM systems without transmit-power constraint [14, 18, 19] and for multiple input multiple output (MIMO) systems [20]. In order to keep the interference at the PU-Rx below a desired level, these papers assumed that the SU-Tx is fully aware of the channel from the SU-Tx to the PU-Rx. However, compared to the intra-system channel state information (CSI) between the SU-Tx and the SU receiver (SU-Rx), which is relatively easy to obtain, it would be difficult or even infeasible for the SU-Tx to obtain full inter-system CSI because the primary and secondary systems are usually loosely coupled (i.e., no explicit communication between them). Even if they are tightly coupled, obtaining full inter-system CSI may be a big burden for the SU due to a large amount of feedback overhead. Therefore, assuming only partial CSI between the SU and the PU seems to be a reasonable approach.

The impact of imperfect channel knowledge and capacity maximization problems with partial CSI have been extensively investigated in the non-CR setting (see [21, 22] and references therein). However, these studies are not directly applicable to our CR setting which has two-dimensional channels: intra-system CSI (between SU-Tx and SU-Rx) and inter-system CSI (between SU-Tx and PU-Rx). Zhang et al. [23] investigated a robust cognitive beamforming problem with partial CSI in MISO (multiple input single output) and MIMO environments. Huang et al. [24] studied the resource allocation problem in OFDM-based CR

networks with partial CSI, where the authors assumed partial intra-system CSI and full inter-system CSI. However, as we mentioned above, the partial inter-system CSI is more reasonable rather than the partial intra-system CSI assumption. Furthermore, there are several studies on the capacity analysis of CR network with imperfect channel knowledge in *flat-fading* environment and these studies assumed that the CSI obtained by the secondary user experiences the channel estimation error [25, 26], while we consider the frequency selective fading environment and assume that the secondary user only knows the statistical properties (e.g., average channel gain) of the inter-system CSI.

1.3 Main contributions and organization of the paper

We would like to mention that this paper is an extended version of our own prior work, [1] and [2], each of which only focuses on power allocation policies with either full inter-system CSI¹ or partial inter-system CSI², respectively.

Beyond the algorithmic contribution, this paper has its own in a different perspective because it contains valuable observations that have not been fully understood yet in existing works. Our novel contributions are summarized as follows: (i) Contrary to existing works that only focused on either partial or full inter-system CSI, this paper have investigated power allocation policies under both partial and full CSI, and further compared their performance with each other. (ii) It has been analytically and numerically verified that the average interference-power tolerable PU is superior to the peak interference-power tolerable PU. (iii) We further investigated the effect of channel correlation by adjusting the level of frequency selectivity. (iv) In addition, we have provided an interesting scenario with multiple primary and secondary receivers to see their effect on the performance.

The remainder of this paper is organized as follows. In Sect. 2, we formally describe our system model including our full/partial CSI assumption, and then present an objective and several constraints. In Sects. 3 and 4, we propose optimal and efficient suboptimal power allocation policies with full and partial inter-system CSI, respectively. In Sect. 5, we evaluate and compare the proposed power allocation policies. Finally, we conclude the paper in Sect. 6.

¹ Kang et al. [31] have derived similar results for the case of full inter-system CSI. However, our research has been produced totally independent from them, and moreover, our previous conference paper [1] was presented more than one year earlier.

² There was a follow-up research [36], where the authors have developed a heuristic algorithm to improve the complexity (i.e., running time) of our algorithms in [2] by removing the loop of binary search at the cost of slight loss in performance.

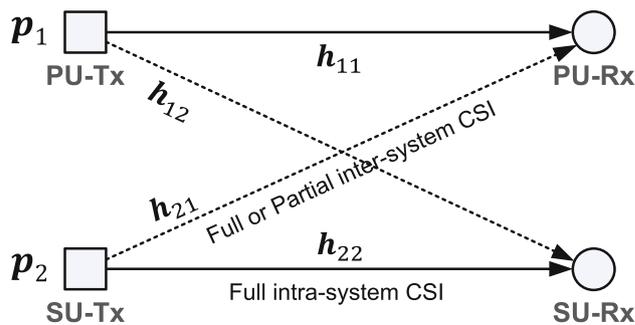


Fig. 1 A channel model for OFDM-based CR networks with full intra-system CSI and full/partial inter-system CSI

2 Model description

2.1 System model

We consider a CR system with a pair of primary transmitter and receiver and a pair of secondary transmitter and receiver, as shown in Fig. 1. The extension to multiple primary and/or secondary receivers will be discussed later on. Both the primary and secondary systems are assumed to be OFDM-based systems using the same spectrum resource for their transmissions. Denote by $\mathcal{N} = \{1, \dots, N\}$ the set of subchannels available.

A channel response from the SU-Tx to the SU-Rx is denoted by $\mathbf{h}_{22} = [h_{22}^1, \dots, h_{22}^N]^T$. Similarly, the channel responses from the PU-Tx to the PU-Rx, from the PU-Tx to the SU-Rx and from the SU-Tx to the PU-Rx are denoted by vectors $\mathbf{h}_{11} = [h_{11}^1, \dots, h_{11}^N]^T$, $\mathbf{h}_{12} = [h_{12}^1, \dots, h_{12}^N]^T$ and $\mathbf{h}_{21} = [h_{21}^1, \dots, h_{21}^N]^T$, respectively. Let $g_{ij}^n = |h_{ij}^n|^2$ denote the channel gain from i to j on subchannel n .

The primary system allocates its power regardless of the secondary’s operation prior to the power allocation of the secondary system. Hence, the SU-Rx is able to measure the amount of interference on each subchannel from the PU-Tx and send this information to the SU-Tx. In addition, assume that the SU-Tx has full CSI for its own link \mathbf{h}_{22} . In other words, it knows instantaneous frequency-selective channel gains g_{22}^n for all subchannels n . On the other hand, measuring the inter-system channel gain g_{21}^n may or may not be so easy as the intra-system channel gain g_{22}^n . In this paper, we take both cases into consideration.

2.1.1 Full inter-system CSI

First, we assume that the SU-Tx has full CSI for the inter-system link \mathbf{h}_{21} . To this end, a brute-force approach is the use of explicit feedback between two systems. An implicit estimation method can be used as well. For example, suppose that the primary system is IEEE 802.16e system operating in time division duplex (TDD) mode, i.e., the

PU-Tx is a base station (BS) and the PU-Rx a mobile station (MS), respectively. The MS transmits channel sounding waveforms on the uplink (MS-to-BS) to enable the BS to estimate the BS-to-MS channel gain under the assumption of TDD reciprocity [27]. The SU-Tx can also overhear this uplink channel sounding signal and measure the channel gain between the MS (PU-Rx) and itself in a similar way. Even though the above method is not applicable, this result still provide the upper-bound performance in the underlay CR networks.

2.1.2 Partial inter-system CSI

Due to the lack of inter-system cooperation, it may not be possible to obtain full inter-system CSI. Instead, we assume that the PU intermittently informs the SU-Tx of only partial CSI about \mathbf{h}_{21} . Based on the assumption that a subchannelization with sufficient interleaving depth is applied, we use an uncorrelated fading channel model [28]. Therefore, in this case, the \mathbf{h}_{21} is a zero-mean complex Gaussian random vector and the channel gains $g_{21}^n = |h_{21}^n|^2$ for all subchannels are independent and identically distributed (i.i.d.) exponential random variables with mean λ_{21} . The partial CSI includes this average channel gain λ_{21} , and we make a further assumption that the channel is time-varying and frequency-selective but the mean remains unchanged until the next feedback.

2.2 Our objective and constraints

In this paper, our goal is to determine the optimal transmit power allocation vector \mathbf{p}_2 of SU-Tx that maximizes the capacity of the SU operating while maintaining the QoS of the PU under the given power budget. To this end, we mathematically formulate the following objective function:

$$\max_{\mathbf{p}_2} \sum_{n \in \mathcal{N}} B \log_2 \left(1 + \frac{1}{\Gamma} \frac{g_{22}^n p_2^n}{g_{12}^n p_1^n + N_0 B} \right), \quad (1)$$

where $\mathbf{p}_1 = [p_1^n, n = 1, 2, \dots, N]$ and $\mathbf{p}_2 = [p_2^n, n = 1, 2, \dots, N]$ are the non-negative power allocation vectors of PU-Tx and SU-Tx, respectively; N_0 denotes the noise power spectral density and B denotes the subchannel bandwidth. Here, a parameter $\Gamma \geq 1$ denotes the signal-to-interference-plus-noise ratio (SINR) gap to ideal Shannon capacity, which is a function of the desired BER (bit error rate), coding gain and noise margin [29].

There are basically two types of constraints on the power allocation. One is a transmit-power constraint at the SU-Tx and the other is an interference-power (outage) constraint at the PU-Rx.

2.2.1 Total transmit-power constraint

This is a conventional constraint which ensures the sum of power allocated over all subchannels is within a power budget P_{max} .

$$\sum_{n \in \mathcal{N}} p_2^n \leq P_{max}. \quad (2)$$

2.2.2 Interference-power constraints

We deal with two types of PUs having different interference tolerability. One is a dumb (peak interference-power tolerable) system that can tolerate a certain amount of peak interference at each subchannel. The other is a more sophisticated (average interference-power tolerable) system that can tolerate the interference as long as the average interference over all subchannels is within a certain threshold. Under the full inter-system CSI assumption, we have the following interference-power constraints.

$$\text{peak: } g_{21}^n p_2^n \leq I_{max}^n, \quad \forall n \in \mathcal{N}, \quad (3)$$

$$\text{average: } \frac{1}{N} \sum_{n \in \mathcal{N}} g_{21}^n p_2^n \leq I_{max}. \quad (4)$$

The constraint (3) ensures that the amount of the interference at each subchannel n is less than I_{max}^n and the constraint (4) ensures that the average amount of the interference received over all subchannels is less than I_{max} . We call I_{max}^n and I_{max} the peak and the average interference temperature levels, respectively. The basic rationale behind this averaging in (4) is that even though there is large interference in some subchannels, small interference in the other subchannels may compensate for the performance of PU in an average sense.

2.2.3 Interference-power outage constraints

Since we cannot strictly guarantee the amount of interference-power under the partial inter-system CSI assumption, we consider the following interference-power *outage* constraints instead.

$$\text{peak: } P_{out}(\mathbf{p}) = \Pr [g_{21}^n p_2^n > I_{max}^n] \leq \epsilon, \quad \forall n \in \mathcal{N}, \quad (5)$$

$$\text{average: } P_{out}(\mathbf{p}) = \Pr \left[\frac{1}{N} \sum_{n \in \mathcal{N}} g_{21}^n p_2^n > I_{max} \right] \leq \epsilon, \quad (6)$$

where the QoS of the PU is guaranteed by keeping an outage probability within a target level ϵ . The outage probability $P_{out}(\cdot)$ is defined as the probability that the interference-power to the PU is greater than a threshold, i.e., the interference-temperature level I_{max}^n or I_{max} .

Remark 1 If PU's own channel is pretty good, i.e., in the bandwidth efficient region, then the PU can tolerate

interference from the SU to a certain extent. Otherwise, in the power efficient region, however, small interference can deteriorate the performance of the PU much. Thus, one could also adaptively change the interference temperature level according to the channel state of the primary system. In this paper, however, we assume that the interference temperature level is given and fixed in a conservative manner, and known at the SU-Tx. We rather focus on the optimal power allocation policy for the SU-Tx for the given interference temperature level. The adaptation of the interference temperature level is beyond the scope of the paper and we leave it as a future study.

3 Case I: full inter-system CSI

We shall start by deriving the optimal power allocation policies under the assumption of full inter-system CSI. For notational simplicity, we denote $p^n \doteq p_2^n$, $g_2^n \doteq g_{22}^n$, $g_1^n \doteq g_{21}^n$ and $v^n \doteq \Gamma(g_{12}^n p_1^n + N_0 B)$, use log instead of \log_2 , and drop B throughout the paper.

3.1 Capacity maximization of SU with peak interference-power tolerable PU: [CM-Peak-Full]

We assume that the PU is a dumb (peak interference-power tolerable) system that can tolerate a certain amount of peak interference at each subchannel. Thus, in this setting, we first consider a capacity maximization problem under the total transmit-power constraint (2) and the *peak* interference-power constraint (3).

[CM-Peak-Full]:

$$\max_{\mathbf{p} \geq 0} \sum_{n \in \mathcal{N}} \log \left(1 + \frac{g_2^n p^n}{v^n} \right) \tag{7}$$

$$\text{subject to } \sum_{n \in \mathcal{N}} p^n \leq P_{max} \tag{8}$$

$$g_1^n p^n \leq I_{max}^n, \forall n \in \mathcal{N}, \tag{9}$$

This problem is a convex optimization problem [30] because a concave function is to be maximized over a convex constraint set and, thus, a unique global solution exists. The constraint (9), which limits the maximum allowable transmit power on the subchannel n to I_{max}^n/g_1^n , is additionally introduced to the classical water-filling problem [34]. Thus, we can obtain the following optimal power allocation policy, so called *capped water-filling* [20]³:

³ It should be noted that some works in literature [20, 31, 32] obtain the similar forms of solutions in different problem settings. The terminology *capped water-filling* comes from the analogy of pouring water into a vessel with both a bumpy ground and a maximum cap.

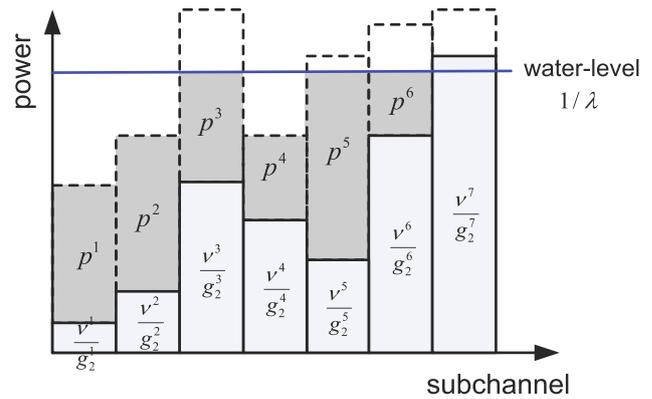


Fig. 2 The graphical interpretation of capped water-filling

Algorithm for **[CM-Peak-Full]**

$$p^n = \left[\frac{1}{\lambda} - \frac{v^n}{g_2^n} \right]_0^{I_{max}^n/g_1^n}, \quad \forall n, \tag{10}$$

where $[z]_a^b \doteq \min[\max[a, z], b]$; λ is a non-negative Lagrange multiplier associated with the total transmit-power constraint (8) and is chosen such that a function

$h(\lambda) \doteq \sum_n p^n(\lambda) - \min[P_{max}, \sum_n \frac{I_{max}^n}{g_1^n}]$ is equal to zero.

Figure 2 shows the graphical interpretation of the capped water-filling. Note that the maximum allowable transmit power on each subchannel is represented as a dotted rectangular box. In order to obtain the solution in (10), we can use an iterative algorithm based on a gradient method which starts from an initial water-level λ , and increases (or decreases) λ with a small step-size if $h(\lambda)$ is greater (or less) than zero until reaching close enough to the optimal solution.

3.2 Capacity maximization of SU with average interference-power tolerable PU: [CM-Avg-Full]

We consider another capacity maximization problem by replacing the peak interference-power constraint (3) with the *average* interference-power constraint (4). In this problem, we assume that the PU operates in a more sophisticated system rather than the dumb system. Thus, the PU can tolerate up to a certain amount of average interference.

[CM-Avg-Full]:

$$\max_{\mathbf{p} \geq 0} \sum_{n \in \mathcal{N}} \log \left(1 + \frac{g_2^n p^n}{v^n} \right) \tag{11}$$

$$\text{subject to } \sum_{n \in \mathcal{N}} p^n \leq P_{max}, \tag{12}$$

$$\frac{1}{N} \sum_{n \in \mathcal{N}} g_1^n p^n \leq I_{max}, \tag{13}$$

This problem is also a convex optimization problem, and thus, a unique global solution exists. The optimal power

allocation policy can be obtained as the following *modified water-filling*:

$$p^n = \left[\frac{1}{\lambda + g_1^n \mu} - \frac{v^n}{g_2^n} \right]^+, \quad \forall n \in \mathcal{N}, \tag{14}$$

where $[z]^+ \triangleq \max[z, 0]$; λ and μ are a non-negative Lagrange multipliers associated with the total transmit-power constraint (12) and the average interference-power constraint (13), respectively. Based on whether the constraint (12) and/or the constraint (13) are active, we can classify the solution into three cases as follows:

1. *Power-limited case* ($\lambda > 0$ and $\mu = 0$):

$p^n = \left[\frac{1}{\lambda} - \frac{v^n}{g_2^n} \right]^+$, where λ is chosen such that the transmit-power constraint (12) holds with equality, $\sum_n p^n = P_{max}$. This case is exactly the same as the classical water-filling solution.

2. *Interference-limited case* ($\lambda = 0$ and $\mu > 0$):

$p^n = \left[\frac{1}{g_1^n \mu} - \frac{v^n}{g_2^n} \right]^+$, where μ is chosen such that interference-power constraint (13) holds with equality, $\frac{1}{N} \sum_n g_1^n p^n = I_{max}$. Using the change of variable $\tilde{p}^n = g_1^n p^n$, this case also can be converted into the classical water-filling solution. $\tilde{p}^n = \left[\frac{1}{\mu} - \frac{v^n g_1^n}{g_2^n} \right]^+$, where μ is chosen such that $\sum_n \tilde{p}^n = N \cdot I_{max}$.

3. *Both-limited case* ($\lambda > 0$ and $\mu > 0$):

$p^n = \left[\frac{1}{\lambda + g_1^n \mu} - \frac{v^n}{g_2^n} \right]^+$, where λ and μ are chosen such that the both constraints hold with equality, $\sum_n p^n = P_{max}$ and $\sum_n g_1^n p^n = N \cdot I_{max}$.

Proposition 1 *If the solution of [CM-Avg-Full] is in the both-limited case, then the optimal Lagrange multipliers λ and μ are always less than or equal to λ_P and μ_I , respectively:*

$$\lambda \leq \lambda_P \text{ and } \mu \leq \mu_I, \tag{15}$$

where λ_P and μ_I are the Lagrange multipliers obtained by assuming the solution is in the power-limited and interference-limited cases, respectively.

Proof We prove this proposition by contradiction. Suppose that $\lambda > \lambda_P$. Since the channel gain g_1^n and μ are positive, consequently $\frac{1}{\lambda + g_1^n \mu} < \frac{1}{\lambda_P}$ holds. Hence, we can obtain the following relationship between p^n and p_P^n that are the optimal power allocation in the both-limited case and the power allocation obtained by assuming the solution is in the power-limited case, respectively:

$$p_P^n = \left[\frac{1}{\lambda_P} - \frac{v^n}{g_2^n} \right]^+ \begin{cases} = p^n = 0, & \text{if } \lambda_P > g_2^n, \\ > p^n \geq 0, & \text{otherwise,} \end{cases} \quad \forall n \in \mathcal{N}. \tag{16}$$

Since there should exist at least one p_P^n with a positive value, we can derive $\sum_n p_P^n > \sum_n p^n$ by summing (16) over all subchannels. This contradicts the fact that the summation of powers in both power-limited and both-limited cases are the same as P_{max} . In a similar way, one can prove $\mu \leq \mu_I$. This completes the proof. \square

The following algorithm describes the detailed procedure for [CM-Avg-Full] with the help of Proposition 1 that allow us to reduce the search space in the STEP 3 and speedup the algorithm.

Algorithm for **[CM-Avg-Full]**

1: *Power-limited case*

$$(\mathbf{p}, \lambda_P) = \text{WATERFILLING}(\{g_2^n\}, P_{max}).$$

If $\frac{1}{N} \sum_n g_1^n p^n < I_{max}$, then go to Finish.

2: *Interference-limited case*

$$(\tilde{\mathbf{p}}, \mu_I) = \text{WATERFILLING}(\{g_2^n/g_1^n\}, N \cdot I_{max}).$$

If $\sum_n p^n = \sum_n \tilde{p}^n/g_1^n < P_{max}$, then go to Finish.

3: *Both-limited case*

Set $\mu_{min} = 0$ and $\mu_{max} = \mu_I$. Repeat the following operations until $\mu_{max} - \mu_{min} \leq \delta$, where δ is a small positive constant which controls the algorithm accuracy.

- Set $\mu = \frac{1}{2}(\mu_{max} + \mu_{min})$ and find the minimum $\lambda \in (0, \lambda_P)$ satisfying

$$\sum_n \left[\frac{1}{\lambda + g_1^n \mu} - \frac{v^n}{g_2^n} \right]^+ = P_{max}.$$

- Obtain power vector \mathbf{p} by putting λ and μ into (14).

- If $\frac{1}{N} \sum_n g_1^n p^n \leq I_{max}$, then $\mu_{max} = \mu$; otherwise, $\mu_{min} = \mu$.

4: Finish: (λ, μ) are the optimal Lagrange multipliers and \mathbf{p} is the optima power allocation

Remark 2 Technically speaking, we can find the optimal power allocation by running only the STEP 3 with arbitrary initial values (λ, μ) . However, in general, finding the optimal solution in the both-limited case requires higher computational complexity than for the conventional water-filling algorithm. This is because we need to determine Lagrange multipliers (λ, μ) in a two-dimensional space. Besides, the optimal solution mostly falls on the power-limited or the interference-limited case rather than the both-limited case. This is why the proposed algorithm first checks whether the optimal solution belongs to the power-limited or the interference-limited case. In either case, a fast conventional water-filling algorithm [33] can be used to obtain the solution. If neither case meets the optimality condition, then we can infer that the optimal solution occurs at the both-limited case.

3.3 Extension to multiple primary and/or secondary receivers

If there are multiple SU-Rxs (say, K), then the best strategy for the SU-Tx is to choose the SU-Rx at each subchannel n having the highest channel $\max_k g_{2,k}^n/v_k^n$ from the SU-Tx to the k -th SU-Rx. If there are multiple PU-Rxs (say, M), then the number of interference-power constraints will increase up to M . Let $g_{1,m}^n$ denote the channel gain from the SU-Tx to the m -th PU-Tx. Consequently, the constraint (9) in [CM-Peak-Full] and the constraint (13) in [CM-Avg-Full] are replaced by the following constraints, respectively:

$$g_{1,m}^n p^n \leq I_{max,m}^n, \quad \forall n, \forall m, \tag{17}$$

$$\frac{1}{N} \sum_{n \in \mathcal{N}} g_{1,m}^n p^n \leq I_{max,m}, \quad \forall m, \tag{18}$$

where $I_{max,m}^n$ and $I_{max,m}$ are the peak and the average interference temperature level for the m -th PU.

Therefore, the optimal power allocation policies for [CM-Peak-Full] and [CM-Avg-Full] can be extended to the scenarios having multiple primary and/or secondary receivers as follows:

[CM-Peak-Full] with multiple PU-Rxs and/or SU-Rxs:

$$p^n = \left[\frac{1}{\lambda} - \frac{1}{\max_k g_{2,k}^n/v_k^n} \right]_0^{\min_m I_{max,m}^n/g_{1,m}^n}, \tag{19}$$

[CM-Avg-Full] with multiple PU-Rxs and/or SU-Rxs:

$$p^n = \left[\frac{1}{\lambda + \sum_m g_{1,m}^n \mu_m} - \frac{1}{\max_k g_{2,k}^n/v_k^n} \right]^+, \tag{20}$$

As expected, the increase of the number of primary receivers M leads to additional constraints limiting the power of the SU-Tx (see the minimum operation in (19) and the summation in (20)), which results in the reduction of capacity. On the other hand, the increase of the number of secondary receivers K gives a multi-user diversity gain [see the maximum operation in (19) and (20)] to the SU-Tx, which results in the increase of capacity.

4 Case II: partial inter-system CSI

In this section, we propose power allocation policies under the assumption of partial inter-system CSI.

4.1 Capacity maximization of SU with peak interference-power tolerable PU:

[CM-Peak-Partial]

The third problem assumes that the PU is peak interference-power tolerable. Thus, in this problem, we attempt to

find an optimal power allocation vector \mathbf{p} for maximizing the capacity under the total transmit-power constraint (2) and the *peak* interference-power *outage* constraint (5).

[CM-Peak-Partial]:

$$\max_{\mathbf{p} \geq 0} \sum_{n \in \mathcal{N}} \log \left(1 + \frac{g_2^n p^n}{v^n} \right) \tag{21}$$

$$\text{subject to } \sum_{n \in \mathcal{N}} p^n \leq P_{max}, \tag{22}$$

$$P_{out}(\mathbf{p}) = Pr[g_1^n p^n > I_{max}^n] \leq \epsilon, \quad \forall n \in \mathcal{N}. \tag{23}$$

Similar to the previous problem [CM-Peak-Full], the problem [CM-Peak-Partial] is the same as the classical water-filling problem [34] except the peak interference-power outage constraint (23). Since g_1^n is assumed to follow an exponential distribution, we can rewrite this constraint (23) as follows:

$$p^n \leq \frac{I_{max}^n}{F_E^{-1}(1 - \epsilon)}, \quad \forall n \in \mathcal{N}, \tag{24}$$

where $F_E^{-1}(\cdot)$ is the inverse cumulative density function (CDF) of an exponential distribution with the mean λ_{21} . It is worthwhile to mention that $F_E^{-1}(1 - \epsilon)$ can be interpreted as *effective channel gain*. The modified constraint (24), which limits the maximum allowable transmit power on each subchannel, is additionally introduced into the classical water-filling problem. Thus, we can obtain the following optimal power allocation policy.

Algorithm for [CM-Peak-Partial]

$$p^n = \left[\frac{1}{\mu} - \frac{v^n}{g_2^n} \right]_0^{\min_m I_{max,m}^n/g_{1,m}^n}, \quad \forall n \in \mathcal{N}, \tag{25} \text{ where}$$

$[z]_a^b \triangleq \min[\max[a, z], b]$; μ is a non-negative Lagrange multiplier associated with the total transmit-power constraint (22) and is chosen such that

$$\sum_{n \in \mathcal{N}} p^n = \min \left[P_{max}, \sum_{n \in \mathcal{N}} \frac{I_{max}^n}{F_E^{-1}(1 - \epsilon)} \right]. \tag{26}$$

4.2 Capacity maximization of SU with average interference-power tolerable PU:

[CM-Avg-Partial]

In our final problem, we try to find an optimal power allocation vector \mathbf{p} for maximizing the capacity under the total transmit-power constraint (2) and the *average* interference-power *outage* constraint (6).

[CM-Avg-Partial]:

$$\max_{\mathbf{p} \geq 0} \sum_{n \in \mathcal{N}} \log \left(1 + \frac{g_2^n p^n}{v^n} \right) \tag{27}$$

$$\text{subject to } \sum_{n \in \mathcal{N}} p^n \leq P_{max}, \tag{28}$$

$$P_{out}(\mathbf{p}) = \Pr \left[\frac{1}{N} \sum_{n \in \mathcal{N}} g_1^n p^n > I_{max} \right] \leq \epsilon. \tag{29}$$

To deal with the problem [CM-Avg-Partial], let us introduce random variables $X_n = p^n g_1^n$ for all subchannels n , which are independently exponential distributed with mean $p^n \lambda_{21}$, and X denotes the sum of these random variables. Then, the average interference-power outage constraint (29) can be rewritten as

$$\Pr \left[X = \sum_{n \in \mathcal{N}} X_n > N \cdot I_{max} \right] \leq \epsilon. \tag{30}$$

To further examine this constraint (30), it is necessary to know the distribution of X . If the transmit power is equally allocated to all the subchannels, i.e., $p^n = p$ for all $n \in \mathcal{N}$, then X follows an Erlang distribution (the sum of several i.i.d. exponential variables), $X \sim \text{Erlang}(N, 1/(p\lambda_{21}))$. Hence, the upper bound power p satisfying the outage constraint can be easily found.

However, in general, the power allocation at the SU-Tx is not even in order to exploit the frequency-selectivity of the channel. Since it is hard to explicitly determine the distribution of X for the general power allocation, we use the Gaussian approximation based on the *Lyapunov's central limit theorem (CLT)* [35]. In order to apply the Lyapunov's CLT, the following *Lyapunov condition* should be satisfied:

$$\lim_{N \rightarrow \infty} \frac{(\sum_{n=1}^N r_n^3)^{\frac{1}{3}}}{(\sum_{n=1}^N \sigma_n^2)^{\frac{1}{2}}} = 0, \tag{31}$$

where r_n is defined as the third central moment of the random variable X_n , i.e., $E[(X_n - m_n)^3]$; m_n and σ_n^2 represent the finite mean and variance of the exponential distributed random variable X_n , respectively. Please refer to Appendix for the detailed proof of this condition.

Thus, for a large number of subchannels, X can be approximated as a normally distributed random variable with mean m and variance σ^2 :

$$m \simeq \sum_n p_n \lambda_{21} \quad \text{and} \quad \sigma^2 \simeq \sum_n (p_n \lambda_{21})^2. \tag{32}$$

Thus, we can rewrite the constraint (30) as:

$$P_{out}(\mathbf{p}) = 1 - F_N(NI_{max}) \tag{33}$$

$$= \frac{1}{2} \text{erfc} \left(\frac{NI_{max} - m}{\sqrt{2}\sigma} \right) \leq \epsilon, \tag{34}$$

where $F_N(\cdot)$ is the CDF of a normal distribution with mean m and variance σ^2 , and $\text{erfc}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-t^2} dt$.

If a power allocation is given, then we can simply check whether it satisfies the outage constraint (34) or not. Unfortunately, however, it is difficult to solve the problem [CM-Avg-Full] simultaneously considering both constraints (28) and (34) because (34) has a very complicated form. Therefore, we alternatively develop a suboptimal power allocation algorithm, which repeatedly (however, it is fast because it requires only a few iterations based on binary search.) solves a subproblem having only a transmit-power constraint using the classical water-filling algorithm and then adjusts the available transmit power \bar{P} until the desired outage probability is achieved. The following Proposition 2 tells us that the outage probability is a strictly increasing function of \bar{P} , and thus we can determine a unique \bar{P} by using *binary search* method.

Proposition 2 *The $P_{out}(\mathbf{p})$ is a strictly increasing function of the available transmit power \bar{P} if the conventional WATERFILLING(\bar{P}) is applied, i.e., $p^n = [1/\mu - v^n/g_2^n]^+$ for all subchannels $n \in \mathcal{N}$, where μ satisfies $\sum_{n \in \mathcal{N}} p^n = \bar{P}$.*

Proof Due to the property of the water-filling algorithm, if the available transmit power \bar{P} increases, then p^n does not decrease for any subchannel n and at least more than one p_n increase. Accordingly, both m and σ^2 in (32) increase as well. Since the $\text{erfc}(\cdot)$ is a decreasing function, $P_{out}(\mathbf{p}) = \frac{1}{2} \text{erfc} \left(\frac{NI_{max} - m}{\sqrt{2}\sigma} \right)$ is a strictly increasing function of \bar{P} . This completes the proof. \square

The following proposed algorithm describes the detailed procedure to find a suboptimal power allocation for [CM-Avg-Partial] with the help of Proposition 2.

Algorithm for [CM-Avg-Partial]

- 1: Initialize:
 - $\bar{P} = P_{max}$ and $\mathbf{p} = \text{WATERFILLING}(P_{max})$.
 - If $P_{out}(\mathbf{p}) > \epsilon + \delta$, then $[a, b] \leftarrow [0, P_{max}]$,
 - Else, go to Finish.
 - 2: Repeat (binary search):
 - $\bar{P} = (a + b)/2$ and $\mathbf{p} = \text{WATERFILLING}(\bar{P})$.
 - If $P_{out}(\mathbf{p}) > \epsilon + \delta$, then $[a, b] \leftarrow [a, \bar{P}]$,
 - Else if $P_{out}(\mathbf{p}) < \epsilon - \delta$, then $[a, b] \leftarrow [\bar{P}, b]$,
 - Else, go to Finish.
 - 3: Finish: \mathbf{p} is a suboptimal power allocation.
-

4.3 Extension to multiple primary and/or secondary receivers

In the multiple (say, K) SU-Rx case, the best strategy for the SU-Tx is to choose the SU-Rx at each subchannel n having the highest channel $g_{2,k}^n/v_k^n$ from the SU-Tx to the

k -th SU-Rx. In the multiple (say, M) PU-Rxs case, although there are multiple interference-power constraints, one constraint with the highest average channel gain $\lambda_{21,m}$ can dominate the others. Therefore, we can obtain the optimal power allocation policies for multiple primary and/or secondary receivers by simply modifying the algorithms for [CM-Peak-Partial] and [CM-Avg-Partial] as follows:

$$\lambda_{21} \leftarrow \max_m \lambda_{21,m}, \quad \text{for } M \text{ PU-Rxs,} \quad (35)$$

$$g_2^n/v^n \leftarrow \max_k g_{2,k}^n/v_k^n, \quad \text{for } K \text{ SU-Rxs.} \quad (36)$$

5 Numerical results

We consider the CR network model as shown in Fig. 1 for our simulations. All channel gains $[g_{11}^n, n \in \mathcal{N}]$, $[g_{12}^n, n \in \mathcal{N}]$, $[g_{21}^n, n \in \mathcal{N}]$ and $[g_{22}^n, n \in \mathcal{N}]$ are independent of each other, and independent and identically distributed (i.i.d.) over all the subchannels. They have unit mean unless specified otherwise. Without loss of generality, the total noise power over the spectrum $(N_0B) \cdot N$ is set to be one and the interference-temperature thresholds are adapted to the level of noise power, i.e., $I_{max} = I_{max}^n = 1/N$ for all subchannels n . We obtain numerical results based on the average performance taken over 10,000 randomly generated channel realizations.

For performance comparison, we further consider simple baseline algorithms based on equal power allocation. In four different capacity maximization problems under either peak or average interference constraint with either full or partial inter-system information, we assume that the transmit power is evenly allocated to all subchannels, i.e., $p^n = p$ for all $n \in \mathcal{N}$. Then, we can easily derive the equal power allocation algorithms as follows.

- [EQ-Peak-Full]: $p = \min \left[\min_{n \in \mathcal{N}} \frac{I_{max}^n}{g_1^n}, \frac{P_{max}}{N} \right]$
- [EQ-Avg-Full]: $p = \min \left[\frac{NI_{max}}{\sum_{n \in \mathcal{N}} g_1^n}, \frac{P_{max}}{N} \right]$
- [EQ-Peak-Partial]: $p = \min \left[\frac{I_{max}^n}{F_E^{-1}(1-\epsilon)}, \frac{P_{max}}{N} \right]$
- [EQ-Avg-Partial]: $p = \min \left[\frac{NI_{max}}{F_{ERL}^{-1}(1-\epsilon)}, \frac{P_{max}}{N} \right]$

where $F_E^{-1}(\cdot)$ is the inverse CDF of an exponential distribution with the mean λ_{21} , and $F_{ERL}^{-1}(\cdot)$ is the inverse CDF of an Erlang distribution, $X \sim Erlang(N, 1/\lambda_{21})$.

5.1 Performance of the power allocation policies with full inter-system CSI

We first compare the performance of the proposed power allocation policies with full CSI when $N = 32$. In addition,

we investigate the impact of the power allocation policies of the PU on the performance of the SU. To this end, either water-filling (WF) or equal power allocation policy is considered as PU’s policy. Thus, the following four combinations [PU’s policy/SU’s policy] are evaluated: [WF/CM-Avg-Full], [WF/CM-Peak-Full], [EQ/CM-Avg-Full] and [EQ/CM-Peak-Full]. For your information, two cases where the PU is limited by either the peak or the average interference-power level are represented by dotted and solid lines in the forthcoming figures, respectively.

Figure 3(a) shows the spectral efficiency of the SU by varying P_{max} . For reference, we include the case without interference constraint (i.e., $I_{max} = I_{max}^n = \infty$ for all $n \in \mathcal{N}$), where in which the spectral efficiency increases logarithmically. In the small P_{max} regime, even though there is a limitation on the amount of interference, the spectral efficiencies for all cases are almost identical because the performance is mainly limited by its own power rather than the interference, i.e., power-limited regime. However, it tends to be eventually saturated as the P_{max} increases, i.e., interference-limited regime. It should be noted that the average interference-power constraint (13) is looser than the peak interference-power constraint (9), and accordingly, allows the SU to have more flexible power allocations. This explains why the spectral efficiency with the former (solid line) is always better than that with the latter (dotted line), e.g., about 20 % in terms of the saturated performance. Interestingly, PU’s WF policy (blue line) maximizing its own channel capacity egoistically improves the performance of the SU as well compared to the EQ policy (red line). This is because the frequency-selective power allocation of the PU brings additional frequency-selectivity to the SU.

Figure 3(b) shows results obtained by varying I_{max} . On the contrary to the previous result in Fig. 3(a), the spectral efficiency starts from the interference-limited regime to the power-limited regime as the I_{max} increases. Other trends can be understood similarly, e.g., the spectral efficiency under the peak and average interference-power constraints and the effect of PU’s policy on the performance of the SU. It is also worthwhile mentioning that our proposed optimal algorithms always perform better than the baseline equal power allocations by exploiting the two-dimensional opportunism of frequency-selectivity.

5.2 Effect of channel correlation

Figure 4 illustrates the effect of channel correlation on the performance of the secondary system. We introduce the following channel model $g_{ij}^n = |h_{ij}^n|^2$ in order to adjust the level of frequency selectivity by a correlation parameter $\alpha \in [0, 1]$.

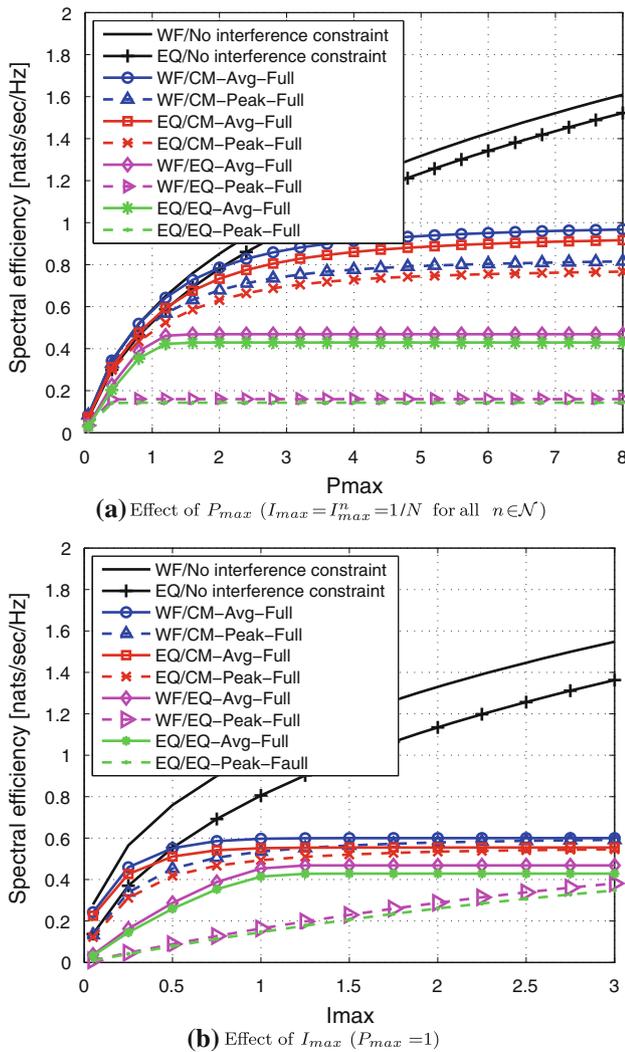


Fig. 3 The performance of the proposed power allocation policies with full CSI. **a** Effect of P_{max} ($I_{max}^n = I_{max}^n = 1/N$ for all $n \in \mathcal{N}$). **b** Effect of I_{max} ($P_{max} = 1$)

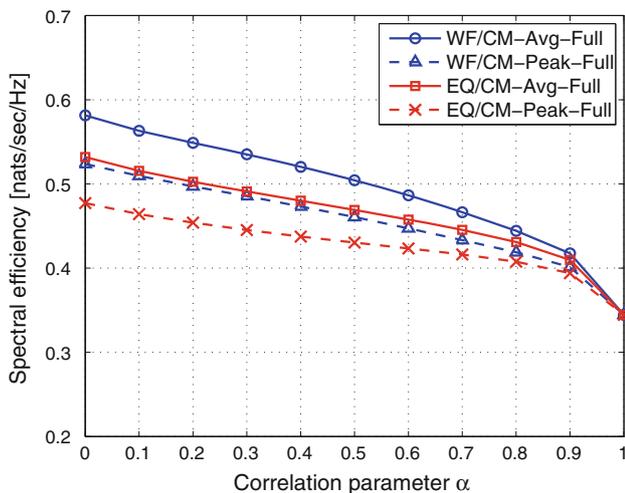


Fig. 4 Effect of correlation ($P_{max} = 1$)

$$h_{ij}^n = \begin{cases} x^n, & \text{if } n = 1, \\ \sqrt{\alpha}h_{ij}^{n-1} + \sqrt{1-\alpha}x^n, & \text{otherwise,} \end{cases} \quad (37)$$

and $\{x^n\}$ are zero-mean unit-variance complex Gaussian random variables and i.i.d. over all subchannels [34]. Note that the frequency selectivity becomes higher as a correlation parameter α decreases. If $\alpha = 0$, then the channel of each subchannel is assumed to be independent of each other. On the other hand, if $\alpha = 1$, then the channel is assumed to be frequency-flat, that is, the same as the single channel setting. For all power allocation policies, the spectral efficiency decreases as the correlation α increases because the correlation reduces the degree of freedom in frequency domain.

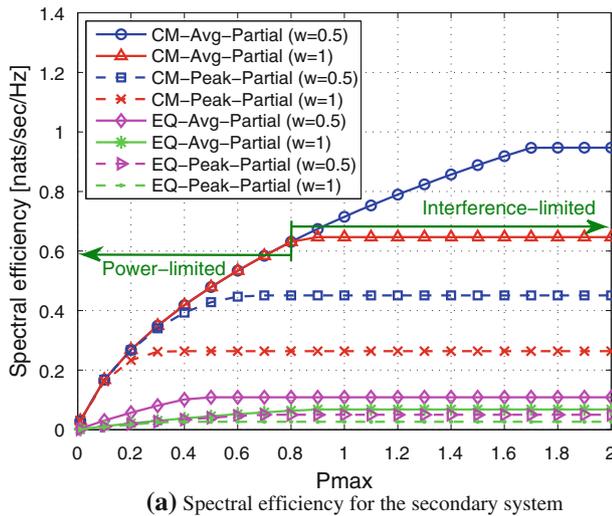
5.3 Performance of the power allocation policies with partial inter-system CSI

We now examine the performance of the proposed power allocation policies with partial CSI by choosing $N = 128$ and $\epsilon = 0.05$. The error tolerance δ for [CM-Avg-Partial] is chosen to be a small value of 10^{-5} . For your information, the number of iterations until the convergence of binary search for [CM-Avg-Partial] was less than 15 times on average.

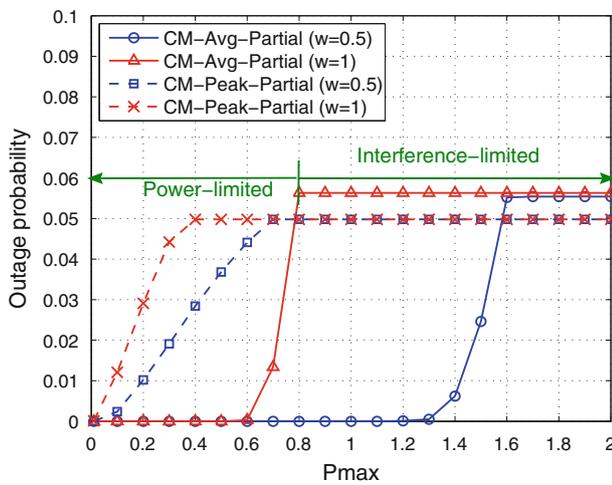
Figure 5(a) shows the spectral efficiency for the SU with respect to the maximal transmit power for different combinations of the ratio $w = \lambda_{21} / \lambda_{22}$ (we fix $\lambda_{22} = 1$ and vary λ_{21}). In the low P_{max} regime, the spectral efficiency increases as the available power increases. On the other hand, when P_{max} is greater than a certain turning point, the spectral efficiency does not further increase because the interference-power outage constraint becomes dominant. We indicate the boundary between power-limited and interference-limited regimes in the case of [CM-Avg-Partial] and $w = 1$ in the middle of figures.

Reducing the ratio w increases the spectral efficiency due to losing interference-power outage constraints (i.e., the PU goes far away from the SU). It is important to highlight that the SU can always obtain the higher spectral efficiency in [CM-Avg-Partial] than [CM-Peak-Partial], e.g., more than two times in terms of the saturated performance. This is because the more sophisticated PU instead of the dumb one gives additional freedom in power allocation to the SU. We may confirm this argument by comparing the interference-power outage constraint of [CM-Avg-Partial] with that of [CM-Peak-Partial]. Since the average interference-power outage constraint (29) is looser than the peak interference-power outage constraint (23) at the same interference-temperature $I_{max}^n = I_{max}$ for all $n \in \mathcal{N}$, more flexible power allocation is possible.

Figure 5(b) shows the outage probability for the PU. In the power-limited regime, the outage probability is much lower than a target $\epsilon = 0.05$. If we keep increasing P_{max}



(a) Spectral efficiency for the secondary system



(b) Outage probability for the primary system

Fig. 5 The performance of the proposed power allocation policies with partial CSI: $N = 128$ and $\epsilon = 0.05$. **a** Spectral efficiency for the secondary system. **b** Outage probability for the primary system

until the interference-limited regime, then the outage probability is saturated to the target. The optimal algorithm for [CM-Peak-Partial] always achieves the exact target requirement, while the suboptimal algorithm for [CM-Avg-Partial] exhibits a small deviation from the target value due to Gaussian approximation error.

5.4 Effect of the number of subchannels on gaussian approximation error

In Figure 6, we investigate the relationship between the total number of subchannels available and Gaussian approximation error. As it can be seen in the figure, the saturated outage probability sticks to the target outage level as the number of subchannels N increases. In other words, the approximation error asymptotically goes to zero. However, if the system does not have the sufficient number

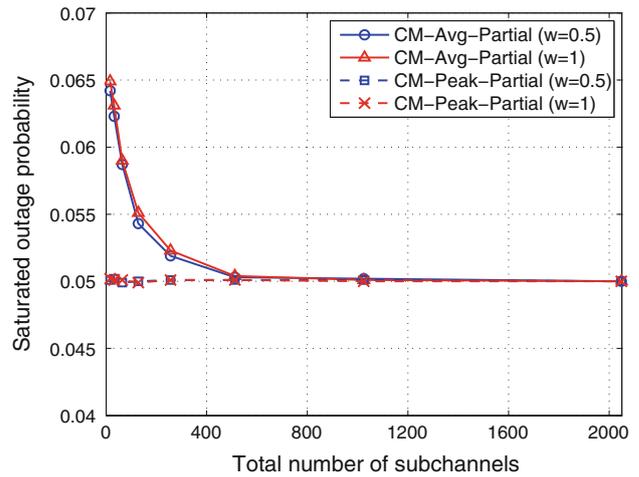


Fig. 6 Saturated outage probability versus the total number of subchannels

of subchannels, a suitable margin on the target error probability will be necessary to make the system robust.

5.5 Performance comparison of power allocation policies with full and partial inter-system CSI

Figure 7 compares the spectral efficiency in the cases of full and partial CSI. We also include the performance without the interference constraint as a reference. As can be observed, the spectral efficiency of power allocation policies with partial CSI in the interference-limited regime increases as the target outage probability ϵ increases because the SU can allocate the power aggressively with looser outage constraint. At the extreme point where ϵ goes to 1, the spectral efficiency with partial CSI becomes equivalent to that without the interference constraint, which might be better than that of algorithms with full CSI.

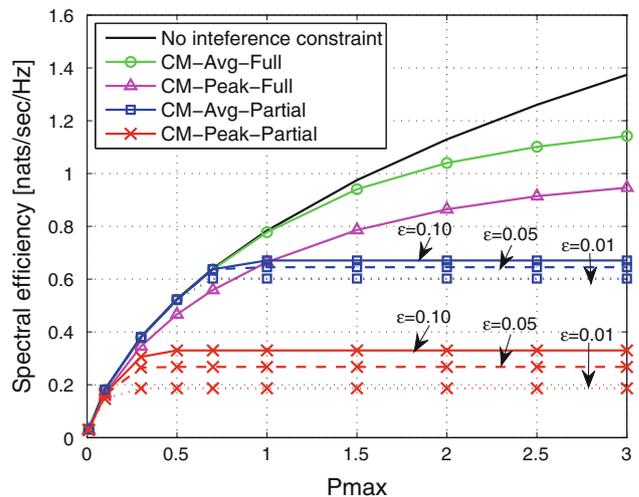


Fig. 7 Performance comparison of policies with full and partial CSI

However, within a reasonable range of outage probability ϵ , e.g., less than 10 %, the spectral efficiency with partial CSI is less than half of the spectral efficiency with full CSI. It is also worthwhile mentioning that PU’s robust tolerability is much beneficial to the SU with partial CSI, e.g., the average interference-power tolerable PU can help to increase the saturated spectral efficiency of the SU by about 20 and 50 % in the cases of full and partial CSI, respectively.

5.6 Effect of multiple primary and/or secondary receivers

Now let us consider the scenarios with multiple primary and secondary receivers. Figure 8 shows the aggregate spectral efficiency of all SUs for the case of [CM-Peak-Full]. We vary both the numbers of primary and secondary receivers, denoted by M and K , respectively, as 1, 2, 4 and 8.

Two observations can be clearly made, which are consistent with the analysis presented in Sect. 3.3. (i) The more SU-RXs K are, the higher aggregate spectral efficiency is likely to be expected due to the multi-user diversity gain. (ii) On the other hand, the more PU-RXs M are, the lower aggregate spectral efficiency is likely to be expected due to the tighter constraints limiting the transmitting power. Although we do not provide the results for the other cases due to limited space, but we also observe similar trends with those in Fig. 8.

6 Conclusion

In this paper, we have investigated both cases where inter-system CSI is fully and partially available to PU-Tx in OFDM-based underlay CR networks. We have also considered two types of PUs having different capabilities: peak and average interference tolerable. Accordingly, we

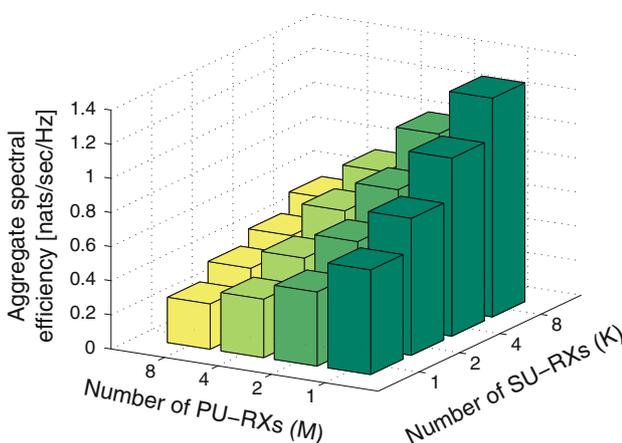


Fig. 8 Effect of multiple primary and/or secondary receivers in the case of [CM-Peak-Full]

have formulated four problems ([CM-Peak-Full], [CM-Avg-Full], [CM-Peak-Partial] and [CM-Avg-Partial]), and proposed optimal and efficient suboptimal power allocation policies for the problems. Through extensive simulations under various scenarios, we have shown that (i) the spectral efficiency achieved by the SU with partial CSI is less than half of what is achieved by the SU with full CSI within a reasonable range of outage probability (e.g., less than 10 %), and (ii) more robust capability of the PU (i.e., average interference-power tolerable) can help to increase the saturated spectral efficiency of the SU by about 20 and 50 % in the cases of full and partial inter-system CSI, respectively. As future work, an extension to more general channel models that include correlation or feedback delay would be an interesting topic.

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Appendix

A Proof of Lyapunov condition

Lyapunov condition :

$$\lim_{N \rightarrow \infty} \frac{(\sum_{n=1}^N r_n^3)^{\frac{1}{3}}}{(\sum_{n=1}^N \sigma_n^2)^{\frac{1}{2}}} = 0, \tag{38}$$

where r_n is defined as the third central moment of random variable X_n , i.e., $E[(X_n - m_n)^3]$; m_n and σ_n^2 are the finite mean and variance of the X_n , respectively.

Proof The third central moment of the random variable X_n can be written as

$$r_n^3 = E[(X_n - m_n)^3] \tag{39}$$

$$= E[X_n^3] - 3m_n E[X_n^2] + 2m_n^3. \tag{40}$$

The random variable $X_n = p^n g_1^n$ is independently exponential distributed with mean $p^n \lambda_{21}$. By plugging the following statistics (41)-(41) for X_n into (40), we can readily obtain the third central moment, $r_n^3 = 2(p^n \lambda_{21})^3$.

mean: $E[X_n] = m_n = p^n \lambda_{21}$,

variance: $Var[X_n] = \sigma_n^2 = (p^n \lambda_{21})^2$,

2th moment: $E[X_n^2] = 2(p^n \lambda_{21})^2$,

3rd moment: $E[X_n^3] = 6(p^n \lambda_{21})^3$.

The power allocation p^n for the subchannel $n \in \mathcal{N} = \{1, 2, \dots, N\}$ is a nonnegative value, whereas the average channel gain λ_{21} is a positive value. To exclude meaningless summations in (38), we define the set of

subchannels with positive power as $\mathcal{N}' \doteq \{n \mid p^n > 0, \forall n \in \mathcal{N}\}$. Since λ_{21} and p^n are finite, there exists positive maximum and minimum values of $p^n \lambda_{21}$ over the set \mathcal{N}' . Let us define the maximum and minimum values as $M = \max_{n \in \mathcal{N}'} p^n \lambda_{21} > 0$ and $m = \min_{n \in \mathcal{N}'} p^n \lambda_{21} > 0$, respectively. Thus, in the (38), the numerator can be upper-bounded and the denominator can be lower-bounded as follows:

$$\left(\sum_{n \in \mathcal{N}'} r_n^3 \right)^{\frac{1}{3}} = \left(\sum_{n \in \mathcal{N}'} 2(p^n \lambda_{21})^3 \right)^{\frac{1}{3}} \leq (|\mathcal{N}'| \cdot 2M^3)^{\frac{1}{3}}, \quad (41)$$

$$(|\mathcal{N}'| \cdot m^2)^{\frac{1}{2}} \leq \left(\sum_{n \in \mathcal{N}'} (p^n \lambda_{21})^2 \right)^{\frac{1}{2}} = \left(\sum_{n \in \mathcal{N}'} \sigma_n^2 \right)^{\frac{1}{2}}, \quad (42)$$

where $|\mathcal{N}'|$ denotes the cardinality of set \mathcal{N}' . Note that as N tends to infinity, $|\mathcal{N}'|$ goes to infinity as well. Using the upper-bound of the numerator (41) and the lower-bound of the denominator (42), we can obtain the Lyapunov condition in the (38).

$$\lim_{N \rightarrow \infty} \frac{\left(\sum_{n=1}^N r_n^3 \right)^{\frac{1}{3}}}{\left(\sum_{n=1}^N \sigma_n^2 \right)^{\frac{1}{2}}} \leq \lim_{N \rightarrow \infty} \frac{(|\mathcal{N}'| \cdot 2M^3)^{\frac{1}{3}}}{(|\mathcal{N}'| \cdot m^2)^{\frac{1}{2}}} \quad (43)$$

$$= \lim_{|\mathcal{N}'| \rightarrow \infty} \frac{2^{\frac{1}{3}} M}{m} \cdot \frac{1}{|\mathcal{N}'|^{\frac{1}{6}}} = 0. \quad (44)$$

This completes the proof. \square

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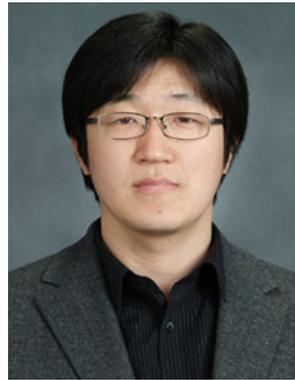
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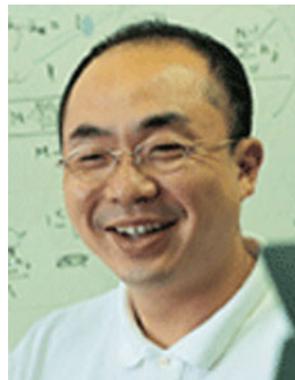
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